COSC 341 – Tutorial 8/9 (Solution)

1. Are the following languages automatic languages? If so, construct an NFA for that language. If not, prove that the language is not automatic.

   (a) $L = \{w | w \text{ is a palindrome over } \{a, b\}\}$

   We will prove that L is not automatic.

   For contradiction we assume that L is automatic.

   Let us consider $z = a^k b a^k \in L, |z| \geq k$. Because of the Pumping Lemma there are u, v and w such that $z = uvw$ with $|u| + |v| \leq k,\ |v| > 0$, and $uv^i w \in L$ for all $i \geq 0$. From $|u| + |v| \leq k,\ |v| > 0$ we can follow that $u = a^{|u|}$, and $v = a^{|v|}$.

   By the Pumping Lemma, uv^2w is element of L as well. It is

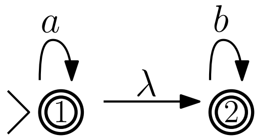
   $$uv^2w = a^{k+|v|} b a^k$$

   with $|v| > 0$. Therefore, uv^2w being an element of L is a contradiction to the definition of L.

   We conclude that L is not an automatic language.

   (b) $L = \{a^n b^m | n, m \in \mathbb{N}\}$

   We prove that L is automatic by giving an automaton accepting L.



   (c) $L = \{a^n b^m | n < m\}$

   We will prove that L is not automatic.

   For contradiction we assume that L is automatic.

   Let us consider $z = a^k b^{k+1} \in L, |z| \geq k$. Because of the Pumping Lemma there are u, v and w such that $z = uvw$ with $|u| + |v| \leq k,\ |v| > 0$, and $uv^i w \in L$ for all $i \geq 0$. From $|u| + |v| \leq k,\ |v| > 0$ we can follow that $u = a^{|u|}$, and $v = a^{|v|}$.

   By the Pumping Lemma, uv^2w is element of L as well. It is

   $$uv^2w = a^{k+|v|} b^{k+1}$$

   with $|v| > 0$. Therefore, uv^2w being an element of L is a contradiction to the definition of L.

   We conclude that L is not an automatic language.

   (d) $L = \{ww | w \in \{a, b\}^*\}$

   We will prove that L is not automatic.

   For contradiction we assume that L is automatic.

   Let us consider $z = a^k b a^k b \in L, |z| \geq k$. Because of the Pumping Lemma there are u, v and w such that $z = uvw$ with $|u| + |v| \leq k,\ |v| > 0$, and $uv^i w \in L$ for all $i \geq 0$. From $|u| + |v| \leq k,\ |v| > 0$ we can follow that $u = a^{|u|}$, and $v = a^{|v|}$.

   By the Pumping Lemma, uv^2w is element of L as well. It is

   $$uv^2w = a^{k+|v|} b a^k b$$

   with $|v| > 0$. Therefore, uv^2w being an element of L is a contradiction to the definition of L.

   We conclude that L is not an automatic language.

Homework

1. Are the following languages automatic languages? If so, construct an NFA for that language. If not, prove that the language is not automatic.

   (a) $L = \{w|w$ has twice as many a's as b's$\}$

   We will prove that L is not automatic.

   For contradiction we assume that L is automatic.

   Let us consider $z = a^{2k}b^k \in L, |z| \geq k$. Because of the Pumping Lemma there are u, v and w such that $z = uvw$ with $|u| + |v| \leq k,\ |v| > 0$, and $uv^iw \in L$ for all $i \geq 0$. From $|u| + |v| \leq k,\ |v| > 0$ we can follow that $u = a^{|u|}$, and $v = a^{|v|}$.

   By the Pumping Lemma, uv^2w is element of L as well. It is

   $$uv^2w = a^{2k+|v|}b^k$$

   with $|v| > 0$. Therefore, uv^2w being an element of L is a contradiction to the definition of L.

   We conclude that L is not an automatic language.

   (b) $L = \{w|w \in \{a, b\}^*,$ the total number of a's and b's is odd$\}$

   We prove that L is not automatic by giving an automaton accepting L.